CBSE Class 10 Mathematics

Important Question

Chapter 8

Introduction to Trigonometry

"The mathematician is fascinated with the marvelous beauty of the forms he constructs, and in their beauty he finds everlasting truth."

1. **If xcosθ – ysinθ = a, xsinθ + ycos θ = b, prove that $x^2+y^2=a^2+b^2$.**

   Ans: xcosθ - y sinθ = a

   xsinθ + y cosθ = b

   Squaring and adding

   $x^2+y^2=a^2+b^2$.

2. **Prove that $sec^2θ+cosec^2θ$ can never be less than 2.**

   Ans: S.T $Sec^2θ + Cosec^2θ$ can never be less than 2.

   If possible let it be less than 2.

   $1 + Tan^2θ + 1 + Cot^2θ < 2$.

   $\Rightarrow 2 + Tan^2θ + Cot^2θ$

   ⇒ (Tanθ + Cotθ)2 < 2.

   Which is not possible.

3. **If** $\sin\varphi=\frac{1}{2}$, **show that** $3\cos\varphi-4\cos^3\varphi=0$

   Ans: Sin ϕ = ½

   $\Rightarrow \phi = 30^0$

   Substituting in place of $\phi = 30^0$. We get 0.

4. **If** $7\sin^2\varphi+3\cos^2\varphi=4\ S.T.$, **show that** $\tan\varphi=\frac{1}{\sqrt{3}}$

   Ans: If $7\sin^2\varphi+3\cos^2\varphi=4\ S.T.\mathrm{Tan}\,\varphi\frac{1}{\sqrt{3}}$

   $7\mathrm{Sin}^2\varphi+3Cos^2\varphi=4(\sin^2\varphi+\mathrm{Cos}^2\varphi)$

   $\Rightarrow 3Sin^2\varphi=\mathrm{Cos}^2\varphi$

   $\Rightarrow \frac{\sin^2\varphi}{\mathrm{Cos}^2\varphi}=\frac{1}{3}$

$\Rightarrow \text{Tan}^2\varphi = \frac{1}{3}$
$\text{Tan}\,\varphi = \frac{1}{\sqrt{3}}$

5. **If** $\cos\varphi + \sin\varphi = \sqrt{2}\cos\varphi$, **prove that** $\cos\varphi - \sin\varphi = \sqrt{2}\sin\varphi$.

Ans: $\text{Cos}\,\varphi + \text{Sin}\,\varphi = \sqrt{2}\,\text{Cos}\,\varphi$
$\Rightarrow (\text{Cos}\,\varphi + \text{Sin}\,\varphi)^2 = 2\text{Cos}^2\varphi$
$\Rightarrow \text{Cos}^2\varphi + \text{Sin}^2\varphi + 2\,\text{Cos}\,\varphi\,\text{Sin}\,\varphi = 2\text{Cos}^2\varphi$
$\Rightarrow \text{Cos}^2\varphi - 2\,\text{Cos}\,\varphi\,\text{Sin}\,\varphi + \text{Sin}^2\varphi = 2\text{Sin}^2\varphi$
$\Rightarrow (\text{Cos}\,\varphi - \text{Sin}\,\varphi)^2 = 2\text{Sin}^2\varphi \begin{bmatrix} \therefore 2\text{Sin}^2\varphi = 2 - 2\text{Cos}^2\varphi \\ 1 - \text{Cos}^2\varphi = \text{Sin}^2\varphi \& 1 - \text{Sin}^2\varphi = \text{Cos}^2\varphi \end{bmatrix}$
Or $\text{Cos}\,\varphi - \text{Sin}\,\varphi = \sqrt{2}\,\text{Sin}\,\varphi$.

6. **If tanA + sinA = m and tanA - sinA = n, show that** $\mathbf{m^2}$**-**$\mathbf{n^2}$ = $= \sqrt{mn}$

Ans: TanA + SinA = m TanA – SinA = n.
$m^2 - n^2 = \sqrt{mn}$.
= m^2 - n^2 = (TanA + SinA)^2 - (TanA - SinA)^2
= 4 TanA SinA
RHS $4\sqrt{mn} = 4\sqrt{(\text{Tan}\,A + \text{Sin}\,A)(\text{Tan}\,A - \text{Sin}\,A)}$
$= 4\sqrt{\text{Tan}^2 A - \text{Sin}^2 A}$
$= 4\sqrt{\frac{\text{Sin}^2 A - \text{Sin}^2 A\text{Cos}^2 A}{\text{Cos}^2 A}}$
$= 4\sqrt{\frac{\text{Sin}^4 A}{\text{Cos}^2 A}}$
$= 4\frac{\text{Sin}^2 A}{\text{Cos}^2 A} = 4\,\text{Tan}\,A\,\text{Sin}\,A$
$\therefore m^2 - n^2 = 4\sqrt{mn}$

7. **If secA=** $x + \frac{1}{4x}$, **prove that secA + tanA=2x or** $\frac{1}{2x}$.

Ans: $\text{Sec}\,\varphi = x + \frac{1}{4x}$
$\Rightarrow \text{Sec}^2\varphi = (x + \frac{1}{4x})^2\ (\sec^2\varphi = 1 + \text{Tan}^2\varphi)$
$\text{Tan}^2\varphi = (x + \frac{1}{4})^2 - 1$
$\text{Tan}^2\varphi = (x - \frac{1}{4})^2$
$\text{Tan}^2\varphi = \pm x - \frac{1}{4x}$
$\text{Sec}\,\varphi + \text{Tan}\,\varphi = x + \frac{1}{4x} \pm x - \frac{1}{4x}$

$= 2x \ or \ \frac{1}{2x}$

8. **If A, B are acute angles and sinA= cosB, then find the value of A+B.**

   Ans: A + B = 90°

9. **a) Solve for ϕ, if tan5ϕ = 1.**

   Ans: $\text{Tan}\, 5\varphi = 1 \Rightarrow \varphi = \frac{45}{5} \Rightarrow \varphi = 9^{\circ}$.

   b) Solve for φ **if** $\frac{\text{Sin}\,\varphi}{1+\text{Cos}\,\varphi} + \frac{1+\text{Cos}\,\varphi}{\text{Sin}\,\varphi} = 4$.

   Ans: $\frac{\text{Sin}\,\varphi}{1+\text{Cos}\,\varphi} + \frac{1+\text{Cos}\,\varphi}{\text{Sin}\,\varphi} = 4$

   $\frac{\text{Sin}^2\varphi+1(\text{Cos}\,\varphi)^2}{\text{Sin}\,\varphi(1+\text{Cos}\,\varphi)} = 4$

   $\frac{\text{Sin}^2\varphi+1+\text{Cos}^2\,\varphi+2\,\text{Cos}\,\varphi}{\text{Sin}\,\varphi+\text{Sin}\,\varphi\,\text{Cos}\,\varphi} = 4$

   $\frac{2+2\,\text{Cos}\,\varphi}{\text{Sin}\,\varphi(1+\text{Cos}\,\varphi)} = 4$

   $\Rightarrow \frac{2+(1+\text{Cos}\,\varphi)}{\text{Sin}\,\varphi(1+\text{Cos}\,\varphi)} = 4$

   $\Rightarrow \frac{2}{\text{Sin}\,\varphi} = 4$

   $\Rightarrow \text{Sin}\,\varphi = \frac{1}{2}$

   $\Rightarrow \text{Sin}\,\varphi = \sin 30$

   $\varphi = 30^{\circ}$

10. **If** $\frac{\cos\alpha}{\cos\beta} = m$ **and** $\frac{\cos\alpha}{\sin\beta} = n$, **show that** $\left(m^2+n^2\right)\cos^2\beta = n^2$

    Ans. $\frac{\text{Cos}\,\alpha}{\text{Cos}\,\beta} = m \ \frac{\text{Cos}\,\alpha}{\text{Sin}\,\beta} = n$

    $\Rightarrow m^2 = \frac{\text{Cos}^2\alpha}{\text{Cos}^2\beta} \ n^2 = \frac{\text{Cos}^2\alpha}{\text{Sin}^2\beta}$

    LHS = $(m^2 + n^2)\ \text{Cos}^2\beta$

    $\left[\frac{\text{Cos}^2\alpha}{Cos^2\beta} + \frac{\text{Cos}^2\alpha}{\text{Sin}^2\beta}\right] \text{Cos}^2\beta$

    $= \text{Cos}^2\alpha\left(\frac{1}{\text{Cos}^2\beta\text{Sin}^2\beta}\right)\text{Cos}^2\beta$

    $= \frac{\text{Cos}^2\alpha}{\text{Sin}^2\beta} = n^2$

    $\Rightarrow (m^2+n^2)\text{Cos}^2\beta = n^2$

11. **If** $7\cos ec\varphi - 3\cot\varphi = 7$,, **prove that** $7\cot\varphi - 3\cos ec\varphi = 3$.

    Ans: $7\,\text{Cos}\,ec\varphi - 2Cot\varphi = 7$

P.T $Cot\varphi - 3\,\text{Cos}\,ec\varphi = 3$

$7\,\text{Cos}\,ec\varphi - 3Cot\varphi = 7$

⇒7Cosecφ - 7=3Cotφ

⇒7(Cosecφ - 1)=3Cotφ

⇒7(Cosecφ-1) (Cosecφ+1)=3Cotφ (Cosecφ+1)

⇒7(Cosec$^2\varphi$-1)=3Cotφ (Cosecφ+1)

⇒7Cot$^2\varphi$.3 Cotφ (Cosecφ+1)

⇒7Cotφ= 3(Cosecφ+1)

7Cotφ-3 Cosecφ=3

12. **2(sin$^6\varphi$+cos$^6\varphi$) – 3(sin$^4\varphi$+cos$^4\varphi$)+1 = 0**

    Ans: (Sin$^2\varphi$)3 + (Cos$^2\varphi$)^3-3 (Sin$^4\varphi$+(Cos$^4\varphi$)+1=0

    Consider (Sin$^2\varphi$)^3 +(Cos$^2\varphi$)^3

    ⇒(Sin$^2\varphi$+Cos$^2\varphi$)^3-3 Sin$^2\varphi$Cos$^2\varphi$ (Sin$^2\varphi$+Cos$^2\varphi$)

    = 1- 3Sin$^2\varphi$ Cos^2

    Sin$^4\varphi$+Cos$^4\varphi$ (Sin$^2\varphi$)^2+(Cos$^2\varphi$)^2

    = (Sin$^2\varphi$+Cos$^2\varphi$)^2-2 Sin$^2\varphi$ Cos^2

    = 1- 2 Sin$^2\varphi$ Cos^2

    = 2(Sin$^6\varphi$+Cos$^6\varphi$)-3(Sin$^4\varphi$+Cos$^4\varphi$) +1

    = 2 (1-3 Sin$^2\varphi$ Cos$^2\varphi$)-3 (1-2 Sin$^2\varphi$+Cos$^2\varphi$)+1

13. **If $\tan\theta = \frac{5}{6}$ & $\theta = \phi = 90^\circ$ what is the value of $\cot\phi$.**

    Ans:$\text{Tan}\,\theta = \frac{5}{6}$ i.e., $\cot\phi = \frac{5}{6}$ Since $\varphi + \theta = 90^\circ$

14. **What is the value of tanφ in terms of sinφ.**

    Ans: $\text{Tan}\,\varphi = \frac{\text{Sin}\,\varphi}{\text{Cos}\,\varphi}$

    $\text{Tan}\,\varphi = \frac{\text{Sin}\,\varphi}{\sqrt{1-\text{Sin}^2\varphi}}$

15. **If Secφ+Tanφ=4 find sin φ, cosφ**

    Ans: Sec φ + Tan φ = 4

    $\frac{1}{\text{Cos}\,\varphi} + \frac{Sin\varphi}{\text{Cos}\,\varphi} = 4$

    $\frac{1+\text{Sin}\,\varphi}{\text{Cos}\,\varphi} = 4$

⇒ apply (C & D)

$$= \frac{(1+\text{Sin}\,\phi)^2+\text{Cos}^2\,\phi}{(1+\text{Sin}\,\phi)^2-\text{Cos}^2\,\phi} = \frac{16+1}{16-1}$$

$$\Rightarrow \frac{2(1+\text{Sin}\,\phi)}{2\,\text{Sin}\,\phi(1+\text{Sin}\,\phi)} = \frac{17}{15}$$

$$\Rightarrow \frac{1}{\text{Sin}\,\varphi} = \frac{17}{15}$$

$$\Rightarrow \text{Sin}\,\varphi = \frac{15}{17}$$

$$\text{Cos}\,\varphi = \sqrt{1-\text{Sin}^2\varphi}$$

$$\sqrt{1-\left(\frac{15}{17}\right)^2} = \frac{8}{17}$$

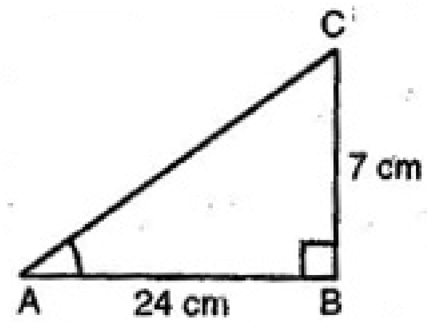
CBSE Class 10 Mathematics
Important Questions
Chapter 8
Introduction to Trigonometry

2 Marks Questions

1. In $\triangle$ ABC, right angled at B, AB = 24 cm, BC = 7 cm. Determine:

(i) $\sin A \cos A$

(ii) $\sin C \cos C$



Ans. Let us draw a right angled triangle ABC, right angled at B.

Using Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

$= (24)^2 + (7)^2 = 576 + 49 = 625$

$\Rightarrow$ AC = 25 cm

(i) $\sin A = \frac{BC}{AC} = \frac{7}{25}$, $\cos A = \frac{AB}{AC} = \frac{24}{25}$

(ii) $\sin C = \frac{AB}{AC} = \frac{24}{25}$, $\cos C = \frac{BC}{AC} = \frac{7}{25}$

2. In adjoining figure, find $\tan P - \cot R$ **:**

Ans. Using Pythagoras theorem,

$PR^2 = PQ^2 + QR^2$

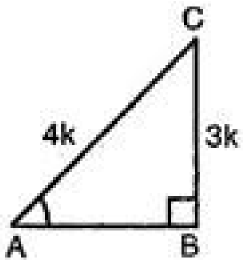
$\Rightarrow (13)^2 = (12)^2 + QR^2$

$\Rightarrow QR^2 = 169 - 144 = 25$

$\Rightarrow QR = 5$ cm

$\therefore \tan P - \cot R = \frac{QR}{PQ} - \frac{QR}{PQ} = \frac{5}{13} - \frac{5}{13} = 0$

3. If $\sin A = \frac{3}{4}$, **calculate** $\cos A$ **and** $\tan A$.



Ans. Given: A triangle ABC in which $\angle B = 90^\circ$

Let BC = $3k$ and AC = $4k$

Then, Using Pythagoras theorem,

AB = $\sqrt{(AC)^2 - (BC)^2} = \sqrt{(4k)^2 - (3k)^2}$

$= \sqrt{16k^2 - 9k^2} = k\sqrt{7}$

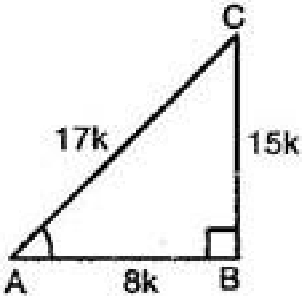
4. Given $15 \cot A = 8$, **find** $\sin A$ **and** $\sec A$

Ans. Given: A triangle ABC in which $\angle B = 90^\circ$

$15 \cot A = 8$

$\Rightarrow \cot A = \frac{8}{15}$

Let AB = $8k$ and BC = $15k$



Then using Pythagoras theorem,

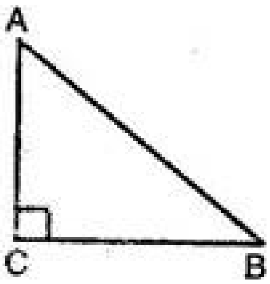
AC = $\sqrt{(AB)^2 + (BC)^2}$ = $\sqrt{(8k)^2 + (15k)^2}$

= $\sqrt{64k^2 + 225k^2}$ = $\sqrt{289k^2}$ = $17k$

$\therefore \sin A = \frac{BC}{AC} = \frac{15k}{17k} = \frac{15}{17}$

$\sec A = \frac{AC}{AB} = \frac{17k}{8k} = \frac{17}{8}$

5. If ∠ And ∠ B are acute angles such that $\cos A = \cos B$, **then show that ∠ A = ∠ B.**



Ans. In right triangle ABC,

$\cos A = \frac{AC}{AB}$ and $\cos B = \frac{BC}{AB}$

But $\cos A = \cos B$ [Given]

$$\Rightarrow \frac{AC}{AB} = \frac{BC}{AB} \Rightarrow AC = BC$$

$$\Rightarrow \angle A = \angle B$$

[Angles opposite to equal sides are equal]

6. State whether the following are true or false. Justify your answer.

(i) The value of $\tan A$ is always less than 1.

(ii) $\sec A = \frac{12}{5}$ for some value of angle A.

(iii) $\cos A$ is the abbreviation used for the cosecant of angle A.

(iv) $\cot A$ is the product of $\cot$ and A.

(v) $\sin\theta = \frac{4}{3}$ for some angle θ.

Ans. (i) False because sides of a right triangle may have any length, so $\tan A$ may have any value.

(ii) True as $\sec A$ is always greater than 1.

(iii) False as $\cos A$ is the abbreviation of cosine A.

(iv) False as $\cot A$ is not the product of 'cot' and A. 'cot' is separated from A has no meaning.

(v) False as $\sin\theta$ cannot be > 1

7. Evaluate:

(i) $\frac{\sin 18^\circ}{\cos 72^\circ}$

(ii) $\frac{\tan 26^\circ}{\cot 64^\circ}$

(iii) $\cos 48^\circ - \sin 42^\circ$

(iv) $\cos ec 31^\circ - \sec 59^\circ$

Ans. Solution:

(i) $\frac{\sin 18^\circ}{\cos 72^\circ} = \frac{\sin\left(90^\circ - 72^\circ\right)}{\cos 72^\circ} = \frac{\cos 72^\circ}{\cos 72^\circ} = 1$

(ii) $\frac{\tan 26^\circ}{\cot 64^\circ} = \frac{\tan\left(90^\circ - 64^\circ\right)}{\cot 64^\circ} = \frac{\cot 64^\circ}{\cot 64^\circ} = 1$

(iii) $\cos 48^\circ - \sin 42^\circ$

$= \cos\left(90^\circ - 42^\circ\right) - \sin 42^\circ$

$= \sin 42^\circ - \sin 42^\circ = 0$

(iv) $\cos ec 31^\circ - \sec 59^\circ$

$= \cos ec\left(90^\circ - 59^\circ\right) - \sec 59^\circ$

$= \sec 59^\circ - \sec 59^\circ = 0$

8. Show that:

(i) $\tan 48^\circ \tan 23^\circ \tan 42^\circ \tan 67^\circ = 1$

(ii) $\cos 38^\circ \cos 52^\circ - \sin 38^\circ \sin 52^\circ = 0$

Ans. (i) L.H.S. $\tan 48^\circ \tan 23^\circ \tan 42^\circ \tan 67^\circ$

$= \tan\left(90^\circ - 42^\circ\right)\tan\left(90^\circ - 67^\circ\right)\tan 42^\circ \tan 67^\circ = \cot 42^\circ \cot 67^\circ \tan 42^\circ \tan 67^\circ$

$= \frac{1}{\tan 42^\circ} \cdot \frac{1}{\tan 67^\circ} \cdot \tan 42^\circ \cdot \tan 67^\circ$ = 1 = R.H.S.

(ii) R.H.S. $\cos 38^\circ \cos 52^\circ - \sin 38^\circ \sin 52^\circ$

$= \cos(90^\circ - 52^\circ) \cdot \cos(90^\circ - 38^\circ) - \sin 38^\circ \cdot \sin 52^\circ$

$= \sin 52^\circ \sin 38^\circ - \sin 38^\circ \sin 52^\circ$ = 0 = R.H.S.

9. If $\tan 2A = \cot(A - 18^\circ)$, where 2A is an acute angle, find the value of A.

Ans. Given: $\tan 2A = \cot(A - 18^\circ)$

$\Rightarrow \cot(90^\circ - 2A) = \cot(A - 18^\circ)$

$\Rightarrow 90^\circ - 2A = A - 18^\circ$

$\Rightarrow -2A - A = -18^\circ - 90^\circ$

$\Rightarrow -3A = -108^\circ$

$\Rightarrow$ A = 36°

10. If $\tan A = \cot B$, prove that $A + B = 90^\circ$.

Ans. Given: $\tan A = \cot B$

$\Rightarrow \cot(90^\circ - A) = \cot B$

$\Rightarrow 90^\circ - A = B$

$\Rightarrow$ A + B = 90°

11. If $\sec 4A = \operatorname{cosec}(A - 20^\circ)$, where 4A is an acute angle, find the value of A.

Ans. Given: $\sec 4A = \operatorname{cosec}(A - 20^\circ)$

$\Rightarrow \operatorname{cosec}(90^\circ - 4A) = \operatorname{cosec}(A - 20^\circ)$

$\Rightarrow 90^\circ - 4A = A - 20^\circ$

$\Rightarrow -4A - A = -20^\circ - 90^\circ$

$\Rightarrow -5A = -110^\circ$

$\Rightarrow$ A = 22°

12. If A, B and C are interior angles of a $\triangle$ ABC, then show that $\sin\left(\frac{B+C}{2}\right) = \cos\frac{A}{2}$.

Ans. Given: A, B and C are interior angles of a $\triangle$ABC.

$\therefore$ A + B + C = 180°

$\Rightarrow \frac{A+B+C}{2} = 90^\circ$

$\Rightarrow \frac{B+C}{2} = 90^\circ - \frac{A}{2}$

$\Rightarrow \sin\left(\frac{B+C}{2}\right) = \sin\left(90^\circ - \frac{A}{2}\right)$

$\Rightarrow \sin\left(\frac{B+C}{2}\right) = \cos\frac{A}{2}$

13. Express $\sin 67^\circ + \cos 75^\circ$ in terms of trigonometric ratios of angles between 0° and 45°.

Ans. $\sin 67^\circ + \cos 75^\circ$

= $\sin(90^\circ - 23^\circ) + \cos(90^\circ - 15^\circ)$

= $\cos 23^\circ + \sin 15^\circ$

14. Express the trigonometric ratios $\sin A$, $\sec A$ and $\tan A$ in terms of $\cot A$

Ans. For $\sin A$,

By using identity $\operatorname{cosec}^2 A - \cot^2 A = 1$

$$\Rightarrow \operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\Rightarrow \frac{1}{\sin^2 A} = 1 + \cot^2 A$$

$$\Rightarrow \sin^2 A = \frac{1}{1 + \cot^2 A}$$

$$\Rightarrow \sin A = \frac{1}{\sqrt{1 + \cot^2 A}}$$

For $\sec A$,

By using identity $\sec^2 A - \tan^2 A = 1$

$$\Rightarrow \sec^2 A = 1 + \tan^2 A$$

$$\Rightarrow \sec^2 A = 1 + \frac{1}{\cot^2 A} = \frac{\cot^2 A + 1}{\cot^2 A}$$

$$\Rightarrow \sec^2 A = \frac{1 + \cot^2 A}{\cot^2 A}$$

$$\Rightarrow \sec A = \frac{\sqrt{1 + \cot^2 A}}{\cot A}$$

For $\tan A$,

$$\tan A = \frac{1}{\cot A}$$

15. Write the other trigonometric ratios of A in terms of $\sec A$

Ans. For $\sin A,$

By using identity, $\sin^2 A+\cos^2 A=1 \Rightarrow \sin^2 A=1-\cos^2 A$

$$\Rightarrow \sin^2 A=1-\frac{1}{\sec^2 A}=\frac{\sec^2 A-1}{\sec^2 A}$$

$$\Rightarrow \sin A=\frac{\sqrt{\sec^2 A-1}}{\sec A}$$

For $\cos A,$

$$\cos A=\frac{1}{\sec A}$$

For $\tan A,$

By using identity $\sec^2 A-\tan^2 A=1 \Rightarrow \tan^2 A=\sec^2 A-1$

$$\Rightarrow \tan A=\sqrt{\sec^2 A-1}$$

For $\cos ecA,$

$$\cos ecA=\frac{1}{\sin A}=\frac{1}{\frac{\sqrt{\sec^2 A-1}}{\sec A}}$$

$$\Rightarrow \cos ecA=\frac{\sec A}{\sqrt{\sec^2 A-1}}$$

For $\cot A,$

$$\cot A=\frac{1}{\tan A}$$

$$\Rightarrow \cot A=\frac{1}{\sqrt{\sec^2 A-1}}$$

16. Evaluate:

(i) $\frac{\sin^2 63° + \sin^2 27°}{\cos^2 17° + \cos^2 73°}$

(ii) $\sin 25° \cos 65° + \cos 25° \sin 65°$

Ans. (i) $\frac{\sin^2 63° + \sin^2 27°}{\cos^2 17° + \cos^2 73°} = \frac{\sin^2 63° + \sin^2 (90° - 63°)}{\cos^2 (90° - 73°) + \cos^2 73°}$

$= \frac{\sin^2 63° + \cos^2 63°}{\sin^2 73° + \cos^2 73°}$

$\left[\because \sin(90° - \theta) = \cos\theta, \cos(90° - \theta) = \sin\theta\right]$

$= \frac{1}{1} = 1 \left[\because \sin^2\theta + \cos^2\theta = 1\right]$

(ii) $\sin 25° \cos 65° + \cos 25° \sin 65°$

$= \sin 25°.\cos(90° - 25°) + \cos 25°.\sin(90° - 25°)$

$= \sin 25°.\sin 25° + \cos 25°.\cos 25°$

$\left[\because \sin(90° - \theta) = \cos\theta, \cos(90° - \theta) = \sin\theta\right]$

$= \sin^2 25° + \cos^2 25° = 1 \left[\because \sin^2\theta + \cos^2\theta = 1\right]$

17. Show that any positive odd integer is of the form6q + 1, or 6q + 3, or 6q + 5, where *q* is some integer.

Ans. Let a be any positive integer and $b = 6$. Then, by Euclid's algorithm,

$a = 6q + r$for some integer $q \geq 0$, and $r = 0, 1, 2, 3, 4, 5$ because $0 \leq r < 6$.

Therefore, $a = 6q$ or $6q + 1$ or $6q + 2$ or $6q + 3$ or $6q + 4$ or $6q + 5$

Also, $6q + 1 = 2 \times 3q + 1 = 2k_1 + 1$, where k_1 is a positive integer

$6q + 3 = (6q + 2) + 1 = 2(3q + 1) + 1 = 2k_2 + 1$, where k_2 is an integer

$6q + 5 = (6q + 4) + 1 = 2(3q + 2) + 1 = 2k_3 + 1$, where k_3 is an integer

Clearly, $6q + 1$, $6q + 3$, $6q + 5$ are of the form $2k + 1$, where k is an integer.

Therefore, $6q + 1$, $6q + 3$, $6q + 5$ are not exactly divisible by 2. Hence, these expressions of numbers are odd numbers.

And therefore, any odd integer can be expressed in the form $6q + 1$, or $6q + 3$,

Or $6q + 5$

18. An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?

Ans. We have to find the HCF(616, 32) to find the maximum number of columns in which they can march.

To find the HCF,we can use Euclid's algorithm.

$616 = 32 \times 19 + 8$

$32 = 8 \times 4 + 0$

The HCF (616, 32) is 8.

Therefore, they can march in 8 columns each.

19. Use Euclid's division lemma to show that the square of any positive integer is either of form $3m$ or $3m + 1$ for some integer m.

[Hint: Let x be any positive integer then it is of the form $3q$, $3q + 1$ or $3q + 2$. Now square each of these and show that they can be rewritten in the form $3m$ or $3m + 1$.]

Ans. Let a be any positive integer and $b = 3$.

Then $a = 3q + r$ for some integer $q \geq 0$

And $r = 0, 1, 2$ because $0 \leq r < 3$

Therefore, $a = 3q$ or $3q + 1$ or $3q + 2$

Or,

$$a^2 = (3q)^2 \, or (3q+1)^2 \, or (3q+2)^2$$
$$a^2 = (9q)^2 \, or 9q^2 + 6q + 1 \, or \, 9q^2 + 12q + 4$$
$$= 3 \times (3q^2) \, or \, 3(3q^2 + 2q) + 1 \, or \, 3(3q^2 + 4q + 1) + 1$$
$$= 3k_1 \, or \, 3k_2 + 1 \, or \, 3k_3 + 1$$

Where k_1, k_2, and k_3 are some positive integers

Hence, it can be said that the square of any positive integer is either of the form $3m$ or $3m + 1$.

CBSE Class 10 Mathematics

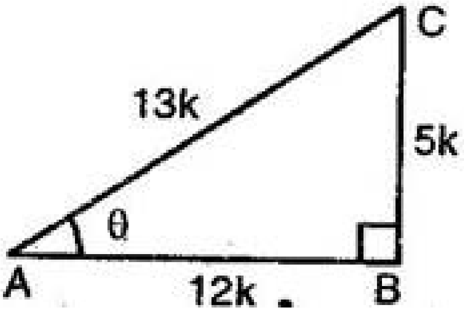
Important Questions

Chapter 8

Introduction to Trigonometry

3 Marks Questions

1. Given $\sec\theta = \frac{13}{12}$, **calculate all other trigonometric ratios.**



Ans. Consider a triangle ABC in which $\angle$ A = θ and $\angle$ B = 90°

Let AB = $12k$ and BC = $5k$

Then, using Pythagoras theorem,

BC = $\sqrt{(AC)^2 - (AB)^2}$

= $\sqrt{(13k)^2 - (12k)^2}$

= $\sqrt{169k^2 - 144k^2}$

= $\sqrt{25k^2}$ = $5k$

$\therefore \sin\theta = \frac{BC}{AC} = \frac{5k}{13k} = \frac{5}{13}$

$\cos\theta = \frac{AB}{AC} = \frac{12k}{13k} = \frac{12}{13}$

$\tan\theta = \frac{BC}{AB} = \frac{5k}{12k} = \frac{5}{12}$

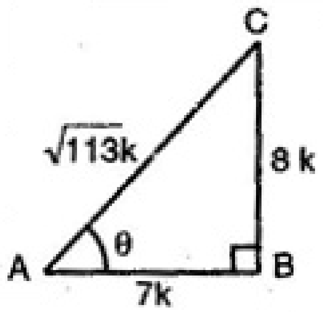
$\cot\theta = \frac{AB}{BC} = \frac{12k}{5k} = \frac{12}{5}$

$\operatorname{cosec}\theta = \frac{AC}{BC} = \frac{13k}{5k} = \frac{13}{5}$

2. If $\cot\theta = \frac{7}{8}$, **evaluate:**

(i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$

(ii) $\cot^2\theta$



Ans. Consider a triangle ABC in which $\angle$A = θ and $\angle$B = 90°

Let AB = $7k$ and BC = $8k$

Then, using Pythagoras theorem,

AC = $\sqrt{(BC)^2+(AB)^2}$ = $\sqrt{(8k)^2+(7k)^2}$

= $\sqrt{64k^2+49k^2}$ = $\sqrt{113k^2}$ = $\sqrt{113}k$

$\therefore \ \sin\theta = \frac{BC}{AC} = \frac{8k}{\sqrt{113}k} = \frac{8}{\sqrt{113}}$

$\cos\theta = \frac{AB}{AC} = \frac{7k}{\sqrt{113}k} = \frac{7}{\sqrt{113}}$

(i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$

= $\frac{1-\sin^2\theta}{1-\cos^2\theta}$

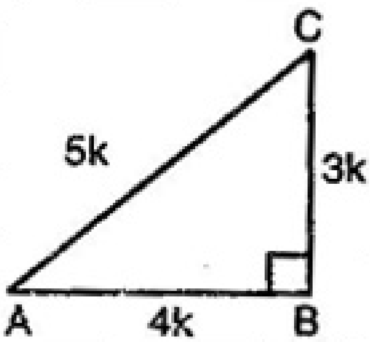
= $\frac{1-\frac{64}{113}}{1-\frac{49}{113}}$

= $\frac{113-64}{113-49}$ = $\frac{49}{64}$

(ii) $\cot^2\theta = \dfrac{\cos^2\theta}{\sin^2\theta}$

$= \dfrac{49/113}{64/113} = \dfrac{49}{64}$

3. If $3\cot A = 4$, check whether $\dfrac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$ or not.



Ans. Consider a triangle ABC in which $\angle$B = 90°.

And $3\cot A = 4 \Rightarrow \cot A = \dfrac{4}{3}$

Let AB = $4k$ and BC = $3k$.

Then, using Pythagoras theorem,

AC = $\sqrt{(BC)^2 + (AB)^2}$

= $\sqrt{(3k)^2 + (4k)^2}$

= $\sqrt{16k^2 + 9k^2}$ = $\sqrt{25k^2}$ = $5k$

$\therefore \sin A = \dfrac{BC}{AC} = \dfrac{3k}{5k} = \dfrac{3}{5}$

$\cos A = \dfrac{AB}{AC} = \dfrac{4k}{5k} = \dfrac{4}{5}$

And $\tan A = \dfrac{BC}{AB} = \dfrac{3k}{4k} = \dfrac{3}{4}$

Now, L.H.S. $\dfrac{1-\tan^2 A}{1+\tan^2 A}$

$= \dfrac{1-\dfrac{9}{16}}{1+\dfrac{9}{16}}$

$= \dfrac{16-9}{16+9} = \dfrac{7}{25}$

R.H.S. $\cos^2 A - \sin^2 A = \left(\dfrac{4}{5}\right)^2 - \left(\dfrac{3}{5}\right)^2$

$= \dfrac{16}{25} - \dfrac{9}{25} = \dfrac{7}{25}$

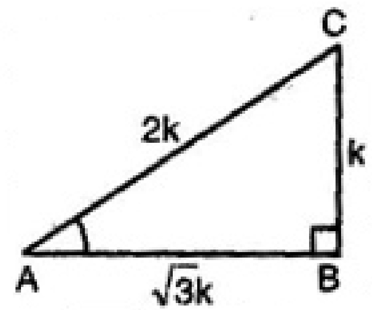
$\because$ L.H.S. = R.H.S.

$\therefore \dfrac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$

4. In Δ ABC right angles at B, if $\tan A = \dfrac{1}{\sqrt{3}}$, find value of:

(i) $\sin A \cos C + \cos A \sin C$

(ii) $\cos A \cos C - \sin A \sin C$



Ans. Consider a triangle ABC in which $\angle$ B = $90°$.

Let BC = k and AB = $\sqrt{3}k$

Then, using Pythagoras theorem,

AC = $\sqrt{(BC)^2 + (AB)^2}$

$= \sqrt{(k)^2 + \left(\sqrt{3}k\right)^2}$

$= \sqrt{k^2 + 3k^2} = \sqrt{4k^2} = 2k$

$\therefore \sin A = \frac{BC}{AC} = \frac{k}{2k} = \frac{1}{2}$

$\cos A = \frac{AB}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$

For $\angle$ C, Base = BC, Perpendicular = AB and Hypotenuse = AC

$\therefore \sin C = \frac{AB}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$

$\cos A = \frac{BC}{AC} = \frac{k}{2k} = \frac{1}{2}$

(i) $\sin A \cos C + \cos A \sin C$

= $\frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$

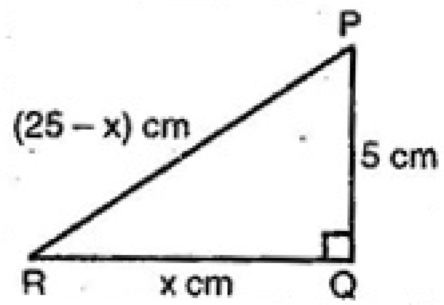
= $\frac{1}{4} + \frac{3}{4} = \frac{4}{4}$ = 1

(ii) $\cos A \cos C - \sin A \sin C$

= $\frac{\sqrt{3}}{2} \times \frac{1}{2} - \frac{1}{2} \times \frac{\sqrt{3}}{2}$

= $\frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0$

5. In Δ PQR, right angled at Q, PR + QR = 25 cm and PQ = 5 cm. Determine the values of $\sin P, \cos P$ and $\tan P$.



Ans. In ΔPQR, right angled at Q.

PR + QR = 25 cm and PQ = 5 cm

Let QR = x cm and PR = $(25 - x)$ cm

Using Pythagoras theorem,

$RP^2 = RQ^2 + QP^2$

$\Rightarrow (25-x)^2 = (x)^2 + (5)^2$

$\Rightarrow 625 - 50x + x^2 = x^2 + 25$

$\Rightarrow -50x = -600$

$\Rightarrow x = 12$

$\therefore$ RQ = 12 cm and RP = 25 – 12 = 13 cm

$\therefore \sin P = \frac{RQ}{RP} = \frac{12}{13}$

$\cos P = \frac{PQ}{RP} = \frac{5}{13}$

And $\tan P = \frac{RQ}{PQ} = \frac{12}{5}$

6. If $\tan(A+B) = \sqrt{3}$ and $\tan(A-B) = \frac{1}{\sqrt{3}}$; $0^\circ < A+B \le 90^\circ$; $A > B$, find A and B.

Ans. (i) False, because $\sin(A+B) = \sin(60^\circ + 30^\circ) = \sin 90^\circ = 1$

And $\sin A + \sin B = \sin 60^\circ + \sin 30^\circ = \frac{\sqrt{3}}{2} + \frac{1}{2} = \frac{\sqrt{3}+1}{2}$

$\therefore \sin(A+B) \ne \sin A + \sin B$

(ii) True, because

θ	0°	30°	45°	60°	90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1

It is clear, the value of $\sin\theta$ increases as θ increases.

(iii) False, because

θ	0°	30°	45°	60°	90°
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0

It is clear, the value of $\cos\theta$ decreases as θ increases

(iv) False as it is only true for $\theta = 45^\circ$.

$$\Rightarrow \sin 45^\circ = \frac{1}{\sqrt{2}} = \cos 45^\circ$$

True, because $\tan 0^\circ = 0$ and $\cot 0^\circ = \frac{1}{\tan 0^\circ}$ = $\frac{1}{0}$ i.e. undefined

7. Choose the correct option. Justify your choice:

(i) $9\sec^2 A - 9\tan^2 A$ =

(A) 1 (B) 9 (C) 8 (D) 0

(ii) $(1+\tan\theta+\sec\theta)(1+\cot\theta-\operatorname{cosec}\theta)$ =

(A) 0 (B) 1 (C) 2 (D) none of these

(iii) $(\sec A+\tan A)(1-\sin A)$ =

(A) $\sec A$ **(B)** $\sin A$ **(C)** $\operatorname{cosec} A$ **(D)** $\cos A$

(iv) $\frac{1+\tan^2 A}{1+\cot^2 A}$ =

(A) $\sec^2 A$ **(B)** -1 **(C)** $\cot^2 A$ **(D) none of these**

Ans. (i) (B) $9\sec^2 A - 9\tan^2 A$ = $9(\sec^2 A - \tan^2 A)$ = 9 x 1 = 9

(ii) (C) $(1+\tan\theta+\sec\theta)(1+\cot\theta-\operatorname{cosec}\theta)$

$$= \left(1+\frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta}\right)\left(1+\frac{\cos\theta}{\sin\theta}-\frac{1}{\sin\theta}\right)$$

$$= \left(\frac{\cos\theta+\sin\theta+1}{\cos\theta}\right)\left(\frac{\sin\theta+\cos\theta-1}{\sin\theta}\right)$$

$$= \frac{(\cos\theta+\sin\theta)^2-(1)^2}{\cos\theta.\sin\theta}$$

$$= \frac{\cos^2\theta+\sin^2\theta+2\cos\theta\sin\theta-1}{\cos\theta.\sin\theta}$$

$$= \frac{1+2\cos\theta\sin\theta-1}{\cos\theta.\sin\theta} \quad [\because \sin^2\theta+\cos^2\theta=1]$$

$$= \frac{2\cos\theta\sin\theta}{\cos\theta.\sin\theta} = 2$$

(iii) (D) $(\sec A+\tan A)(1-\sin A)$

$= \left(\dfrac{1}{\cos A}+\dfrac{\sin A}{\cos A}\right)(1-\sin A)$

$= \left(\dfrac{1+\sin A}{\cos A}\right)(1-\sin A)$

$= \dfrac{1-\sin^2 A}{\cos A} = \dfrac{\cos^2 A}{\cos A} = \cos A$

$\left[\because 1-\sin^2 A=\cos^2 A\right]$

(iv) (D) $\dfrac{1+\tan^2 A}{1+\cot^2 A} = \dfrac{\sec^2 A-\tan^2 A+\tan^2 A}{\operatorname{cosec}^2 A-\cot^2 A+\cot^2 A}$

$= \dfrac{\sec^2 A}{\operatorname{cosec}^2 A} = \dfrac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}}$

$= \dfrac{\sin^2 A}{\cos^2 A} = \tan^2 A$

CBSE Class 10 Mathematics

Important Questions

Chapter 8

Introduction to Trigonometry

4 Marks Questions

1. Express the trigonometric ratios $\sin A$, $\sec A$ and $\tan A$ in terms of $\cot A$

Ans. For $\sin A$,

By using identity $\operatorname{cosec}^2 A - \cot^2 A = 1$

$\Rightarrow \operatorname{cosec}^2 A = 1 + \cot^2 A$

$\Rightarrow \frac{1}{\sin^2 A} = 1 + \cot^2 A$

$\Rightarrow \sin^2 A = \frac{1}{1 + \cot^2 A}$

$\Rightarrow \sin A = \frac{1}{\sqrt{1 + \cot^2 A}}$

For $\sec A$,

By using identity $\sec^2 A - \tan^2 A = 1$

$\Rightarrow \sec^2 A = 1 + \tan^2 A$

$\Rightarrow \sec^2 A = 1 + \frac{1}{\cot^2 A} = \frac{\cot^2 A + 1}{\cot^2 A}$

$\Rightarrow \sec^2 A = \frac{1 + \cot^2 A}{\cot^2 A}$

$$\Rightarrow \sec A = \frac{\sqrt{1+\cot^2 A}}{\cot A}$$

For $\tan A$,

$$\tan A = \frac{1}{\cot A}$$

2. Write the other trigonometric ratios of A in terms of $\sec A$

Ans. For $\sin A$,

By using identity, $\sin^2 A + \cos^2 A = 1$

$$\Rightarrow \sin^2 A = 1 - \cos^2 A$$

$$\Rightarrow \sin^2 A = 1 - \frac{1}{\sec^2 A} = \frac{\sec^2 A - 1}{\sec^2 A}$$

$$\Rightarrow \sin A = \frac{\sqrt{\sec^2 A - 1}}{\sec A}$$

For $\cos A$,

$$\cos A = \frac{1}{\sec A}$$

For $\tan A$,

By using identity $\sec^2 A - \tan^2 A = 1$

$$\Rightarrow \tan^2 A = \sec^2 A - 1$$

$$\Rightarrow \tan A = \sqrt{\sec^2 A - 1}$$

For $\cos ecA$,

$$\operatorname{cosec} A = \frac{1}{\sin A} = \frac{1}{\frac{\sqrt{\sec^2 A - 1}}{\sec A}}$$

$$\Rightarrow \operatorname{cosec} A = \frac{\sec A}{\sqrt{\sec^2 A - 1}}$$

For $\cot A$,

$$\cot A = \frac{1}{\tan A}$$

$$\Rightarrow \cot A = \frac{1}{\sqrt{\sec^2 A - 1}}$$

3. Evaluate:

(i) $\dfrac{\sin^2 63° + \sin^2 27°}{\cos^2 17° + \cos^2 73°}$

(ii) $\sin 25° \cos 65° + \cos 25° \sin 65°$

Ans. (i) $\dfrac{\sin^2 63° + \sin^2 27°}{\cos^2 17° + \cos^2 73°}$

$$= \frac{\sin^2 63° + \sin^2 (90° - 63°)}{\cos^2 (90° - 73°) + \cos^2 73°}$$

$$= \frac{\sin^2 63° + \cos^2 63°}{\sin^2 73° + \cos^2 73°}$$

$$\left[\because \sin\left(90° - \theta\right) = \cos\theta, \cos\left(90° - \theta\right) = \sin\theta\right]$$

$$= \frac{1}{1} = 1 \quad \left[\because \sin^2\theta + \cos^2\theta = 1\right]$$

(ii) $\sin 25° \cos 65° + \cos 25° \sin 65°$

= $\sin 25°.\cos(90° - 25°) + \cos 25°.\sin(90° - 25°)$

= $\sin 25°.\sin 25° + \cos 25°.\cos 25°$

$\left[\because \sin(90° - \theta) = \cos\theta, \cos(90° - \theta) = \sin\theta\right]$

= $\sin^2 25° + \cos^2 25°$ = 1

$\left[\because \sin^2\theta + \cos^2\theta = 1\right]$

= $\sin^2 25° + \cos^2 25°$ = 1

$\left[\because \sin^2\theta + \cos^2\theta = 1\right]$

4. Choose the correct option. Justify your choice:

(i) $9\sec^2 A - 9\tan^2 A$ =

(A) 1 (B) 9 (C) 8 (D) 0

(ii) $(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \operatorname{cosec}\theta)$ =

(A) 0 (B) 1 (C) 2 (D) none of these

(iii) $(\sec A + \tan A)(1 - \sin A)$ =

(A) $\sec A$ **(B)** $\sin A$ **(C)** $\operatorname{cosec} A$ **(D)** $\cos A$

(iv) $\dfrac{1 + \tan^2 A}{1 + \cot^2 A}$ =

(A) $\sec^2 A$ **(B)** -1 **(C)** $\cot^2 A$ **(D) none of these**

Ans. (i) (B) $9\sec^2 A - 9\tan^2 A$

= $9(\sec^2 A - \tan^2 A)$ = 9 x 1 = 9

(ii) (C) $\left(1+\tan\theta+\sec\theta\right)\left(1+\cot\theta-\operatorname{cos}ec\theta\right)$

= $\left(1+\dfrac{\sin\theta}{\cos\theta}+\dfrac{1}{\cos\theta}\right)\left(1+\dfrac{\cos\theta}{\sin\theta}-\dfrac{1}{\sin\theta}\right)$

= $\left(\dfrac{\cos\theta+\sin\theta+1}{\cos\theta}\right)\left(\dfrac{\sin\theta+\cos\theta-1}{\sin\theta}\right)$

= $\dfrac{\left(\cos\theta+\sin\theta\right)^2-(1)^2}{\cos\theta.\sin\theta}$

= $\dfrac{\cos^2\theta+\sin^2\theta+2\cos\theta\sin\theta-1}{\cos\theta.\sin\theta}$

= $\dfrac{1+2\cos\theta\sin\theta-1}{\cos\theta.\sin\theta}$

$\left[\because \sin^2\theta+\cos^2\theta=1\right]$

= $\dfrac{2\cos\theta\sin\theta}{\cos\theta.\sin\theta}$ = 2

(iii) (D) $\left(\sec A+\tan A\right)\left(1-\sin A\right)$

= $\left(\dfrac{1}{\cos A}+\dfrac{\sin A}{\cos A}\right)\left(1-\sin A\right)$

= $\left(\dfrac{1+\sin A}{\cos A}\right)\left(1-\sin A\right)$

= $\dfrac{1-\sin^2 A}{\cos A}$ = $\dfrac{\cos^2 A}{\cos A}$ = $\cos A$

$\left[\because 1-\sin^2 A=\cos^2 A\right]$

(iv) (D) $\dfrac{1+\tan^2 A}{1+\cot^2 A}$

$= \dfrac{\sec^2 A - \tan^2 A + \tan^2 A}{\operatorname{cosec}^2 A - \cot^2 A + \cot^2 A} = \dfrac{\sec^2 A}{\operatorname{cosec}^2 A}$

$= \dfrac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}} = \dfrac{\sin^2 A}{\cos^2 A} = \tan^2 A$

5. Prove the following identities, where the angles involved are acute angles for which the expressions are defined:

(i) $(\operatorname{cosec}\theta - \cot\theta)^2 = \dfrac{1-\cos\theta}{1+\cos\theta}$

(ii) $\dfrac{\cos A}{1+\sin A} + \dfrac{1+\sin A}{\cos A} = 2\sec A$

(iii) $\dfrac{\tan\theta}{1-\cot\theta} + \dfrac{\cot\theta}{1-\tan\theta} = 1+\sec\theta\operatorname{cosec}\theta$

(iv) $\dfrac{1+\sec A}{\sec A} = \dfrac{\sin^2 A}{1-\cos A}$

(v) $\dfrac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \operatorname{cosec} A + \cot A$ **, using the identity** $\operatorname{cosec}^2 A = 1+\cot^2 A$

(vi) $\sqrt{\dfrac{1+\sin A}{1-\sin A}} = \sec A + \tan A$

(vii) $\dfrac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$

(viii) $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$

(ix) $(\operatorname{cosec} A - \sin A)(\sec A - \cos A) = \dfrac{1}{\tan A + \cot A}$

(x) $\left(\dfrac{1+\tan^2 A}{1+\cot^2 A}\right) = \left(\dfrac{1-\tan A}{1-\cot A}\right)^2 = \tan^2 A$

Ans. Proof:

(i) L.H.S. $(\operatorname{cosec}\theta - \cot\theta)^2$

$= \operatorname{cosec}^2\theta + \cot^2\theta - 2\operatorname{cosec}\theta\cot\theta$

$= \dfrac{1}{\sin^2\theta} + \dfrac{\cos^2\theta}{\sin^2\theta} - 2\times\dfrac{1}{\sin\theta}.\dfrac{\cos\theta}{\sin\theta}$

$= \dfrac{1+\cos^2\theta}{\sin^2\theta} - \dfrac{2\cos\theta}{\sin^2\theta}$

$= \dfrac{1+\cos^2\theta - 2\cos\theta}{\sin^2\theta}$

$= \dfrac{(1-\cos\theta)^2}{\sin^2\theta}$

$\left[\because a^2+b^2-2ab=(a-b)^2\right]$

$= \dfrac{(1-\cos\theta)(1-\cos\theta)}{1-\cos^2\theta}$

$= \dfrac{(1-\cos\theta)(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}$

$= \dfrac{1-\cos\theta}{1+\cos\theta}$ = R.H.S.

(ii) L.H.S. $\frac{\cos A}{1+\sin A}+\frac{1+\sin A}{\cos A}$

$=\frac{\cos^2\theta+1+\sin^2\theta+2\sin A}{(1+\sin A)\cos A}$

$=\frac{\cos^2\theta+\sin^2\theta+1+2\sin A}{(1+\sin A)\cos A}$

$=\frac{1+1+2\sin A}{(1+\sin A)\cos A}$

$\left[\because \sin^2\theta+\cos^2\theta=1\right]$

$=\frac{2+2\sin A}{(1+\sin A)\cos A}=\frac{2(1+\sin A)}{(1+\sin A)\cos A}$

$=\frac{2}{\cos A}=2\sec A$ = R.H.S

(iii) L.H.S. $\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}$

$=\frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}}+\frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}}$

$=\frac{\sin\theta}{\cos\theta}\times\frac{\sin\theta}{\sin\theta-\cos\theta}+\frac{\cos\theta}{\sin\theta}\times\frac{\cos\theta}{\cos\theta-\sin\theta}$

$=\frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)}+\frac{\cos^2\theta}{\sin\theta(\cos\theta-\sin\theta)}$

$=\frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)}-\frac{\cos^2\theta}{\sin\theta(\sin\theta-\cos\theta)}$

$$= \frac{\sin^3\theta - \cos^3\theta}{\sin\theta\cos\theta(\sin\theta - \cos\theta)}$$

$$= \frac{(\sin\theta - \cos\theta)(\sin^2\theta + \cos^2\theta + \sin\theta\cos\theta)}{\sin\theta\cos\theta(\sin\theta - \cos\theta)}$$

$$\left[\because a^3 - b^3 = (a-b)(a^2 + b^2 + ab)\right]$$

$$= \frac{1 + \sin\theta\cos\theta}{\sin\theta\cos\theta} \quad \left[\because \sin^2\theta + \cos^2\theta = 1\right]$$

$$= \frac{1}{\sin\theta\cos\theta} + 1 = 1 + \frac{1}{\sin\theta\cos\theta}$$

$$= 1 + \sec\theta\cos ec\theta$$

(iv) L.H.S. $\frac{1+\sec A}{\sec A} = \frac{1 + \frac{1}{\cos A}}{\frac{1}{\cos A}}$

$$= \frac{\cos A + 1}{\cos A} \times \frac{\cos A}{1} = 1 + \cos A$$

$$= 1 + \cos A \times \frac{1 - \cos A}{1 - \cos A}$$

$$= \frac{1 - \cos^2 A}{1 - \cos A}$$

$$= \frac{\sin^2 A}{1 - \cos A} = \text{R.H.S.}$$

(v) L.H.S. $\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1}$

Dividing all terms by $\sin A$,

$= \dfrac{\cot A - 1 + \operatorname{cosec} A}{\cot A + 1 - \operatorname{cosec} A} = \dfrac{\cot A + \operatorname{cosec} A - 1}{\cot A - \operatorname{cosec} A + 1}$

$= \dfrac{(\cot A + \operatorname{cosec} A) - (\operatorname{cosec}^2 A - \cot^2 A)}{(1 + \cot A - \operatorname{cosec} A)}$

$= \dfrac{(\cot A + \operatorname{cosec} A) + (\cot^2 A - \operatorname{cosec}^2 A)}{(1 + \cot A - \operatorname{cosec} A)}$

$= \dfrac{(\cot A + \operatorname{cosec} A)(1 + \cot A - \operatorname{cosec} A)}{(1 + \cot A - \operatorname{cosec} A)}$

$= \cot A + \operatorname{cosec} A$ = R.H.S.

(vi) L.H.S. $\sqrt{\dfrac{1+\sin A}{1-\sin A}}$

$= \sqrt{\dfrac{1+\sin A}{1-\sin A}} \times \sqrt{\dfrac{1+\sin A}{1+\sin A}}$

$= \sqrt{\dfrac{(1+\sin A)^2}{1-\sin^2 A}}$

$[\because (a+b)(a-b) = a^2 - b^2]$

$= \sqrt{\dfrac{(1+\sin A)^2}{\cos^2 A}}$ $[\because 1 - \sin^2\theta = \cos^2\theta]$

$= \dfrac{1+\sin A}{\cos A} = \dfrac{1}{\cos A} + \dfrac{\sin A}{\cos A}$

$= \sec A + \tan A$ = R.H.S.

(vii) L.H.S. $\dfrac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \dfrac{\sin\theta(1 - 2\sin^2\theta)}{\cos\theta(2\cos^2\theta - 1)}$

$= \dfrac{\sin\theta\left(1-2\sin^2\theta\right)}{\cos\theta\left[2\left(1-\sin^2\theta\right)-1\right]}$ $\left[\because 1-\sin^2\theta=\cos^2\theta\right]$

$= \dfrac{\sin\theta\left(1-2\sin^2\theta\right)}{\cos\theta\left(2-2\sin^2\theta-1\right)}$

$= \dfrac{\sin\theta\left(1-2\sin^2\theta\right)}{\cos\theta\left(1-2\sin^2\theta\right)} = \dfrac{\sin\theta}{\cos\theta} = \tan\theta$ = R.H.S

(viii) L.H.S. $\left(\sin A+\operatorname{cosec} A\right)^2+\left(\cos A+\sec A\right)^2$

$= \left(\sin A+\dfrac{1}{\sin A}\right)^2+\left(\cos A+\dfrac{1}{\cos A}\right)^2$

$= \sin^2 A+\dfrac{1}{\sin^2 A}+2.\sin A.\dfrac{1}{\sin A}+\cos^2 A+\dfrac{1}{\cos^2 A}+2.\cos A.\dfrac{1}{\cos A}$

$= 2+2+\sin^2 A+\cos^2 A+\dfrac{1}{\sin^2 A}+\dfrac{1}{\cos^2 A}$

$= 4+1+\dfrac{1}{\sin^2 A}+\dfrac{1}{\cos^2 A}$

$= 5+\operatorname{cosec}^2 A+\sec^2 A$

$= 5+1+\cot^2 A+1+\tan^2 A$

$\left[\because \operatorname{cosec}^2\theta=1+\cot^2\theta, \sec^2\theta=1+\tan^2\theta\right]$

$= 7+\tan^2 A+\cot^2 A$

= R.H.S.

(ix) L.H.S. $\left(\operatorname{cosec} A-\sin A\right)\left(\sec A-\cos A\right)$

$$= \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)$$

$$= \left(\frac{1-\sin^2 A}{\sin A}\right)\left(\frac{1-\cos^2 A}{\cos A}\right)$$

$$= \frac{\cos^2 A}{\sin A} \times \frac{\sin^2 A}{\cos A} = \sin A . \cos A$$

$$= \frac{\sin A . \cos A}{\sin^2 A + \cos^2 A} \quad \left[\because \sin^2\theta + \cos^2\theta = 1\right]$$

Dividing all the terms by $\sin A . \cos A$,

$$= \frac{\frac{\sin A . \cos A}{\sin A . \cos A}}{\frac{\sin^2 A}{\sin A . \cos A} + \frac{\cos^2 A}{\sin A . \cos A}} = \frac{1}{\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}}$$

$$= \frac{1}{\tan A + \cot A} = \text{R.H.S.}$$

(x) L.H.S. $\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right)$

$$= \frac{\sec^2 A}{\cos ec^2 A} \quad \left[\because 1+\tan^2\theta = \sec^2\theta, 1+\cot^2\theta = \cos ec^2\theta\right]$$

$$= \frac{1}{\cos^2 A} \times \frac{\sin^2 A}{1} = \tan^2 A = \text{R.H.S.}$$

Now, Middle side = $\left(\frac{1-\tan A}{1-\cot A}\right)^2 = \left(\frac{1-\tan A}{1-\frac{1}{\tan A}}\right)^2$

$$= \left(\frac{\dfrac{1-\tan A}{\tan A - 1}}{\tan A} \right)^2$$

$$= \left(\frac{\dfrac{1-\tan A}{-(1-\tan A)}}{\tan A} \right) = (-\tan A)^2 = \tan^2 A = \text{R.H.S.}$$

6. Use Euclid's division algorithm to find the HCFof:

(i) 135 and 225

(ii) 196 and 38220

(iii) 867 and 255

Ans. (i) 135 and 225

We have 225 > 135,

So, we apply the division lemma to 225 and 135 to obtain

225 = 135 × 1 + 90

Here remainder 90 ≠ 0, we apply the division lemma again to 135 and 90 to obtain

135 = 90 × 1 + 45

We consider the new divisor 90 and new remainder 45≠ 0, and apply the division lemma to obtain

90 = 2 × 45 + 0

Since that time the remainder is zero, the process get stops.

The divisor at this stage is 45

Therefore, the HCF of 135 and 225 is 45.

(ii) 196 and 38220

We have 38220 > 196,

So, we apply the division lemma to 38220 and 196 to obtain

$38220 = 196 \times 195 + 0$

Since we get the remainder is zero, the process stops.

The divisor at this stage is 196,

Therefore, HCF of 196 and 38220 is 196.

(iii) 867 and 255

We have 867 > 255,

So, we apply the division lemma to 867 and 255 to obtain

$867 = 255 \times 3 + 102$

Here remainder $102 \neq 0$, we apply the division lemma again to 255 and 102 to obtain

$255 = 102 \times 2 + 51$

Here remainder $51 \neq 0$, we apply the division lemma again to 102 and 51 to obtain

$102 = 51 \times 2 + 0$

Since we get the remainder is zero, the process stops.

The divisor at this stage is 51,

Therefore, HCF of 867 and 255 is 51.

7. Evaluate:

(i) $\sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ$

(ii) $2\tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$

(iii) $\dfrac{\cos 45^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ}$

(iv) $\dfrac{\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ}$

(v) $\dfrac{5\cos^2 60^\circ + 4\sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ}$

Ans. (i) $\sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ = \dfrac{\sqrt{3}}{2} \times \dfrac{\sqrt{3}}{2} + \dfrac{1}{2} \times \dfrac{1}{2}$

$= \dfrac{3}{4} + \dfrac{1}{4} = \dfrac{4}{4} = 1$

(ii) $2\tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ = 2(1)^2 + \left(\dfrac{\sqrt{3}}{2}\right)^2 - \left(\dfrac{\sqrt{3}}{2}\right)^2$

$= 2 + \dfrac{3}{4} - \dfrac{3}{4} = 2$

(iii) $\dfrac{\cos 45^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ}$

$= \dfrac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}} + 2} = \dfrac{\frac{1}{\sqrt{2}}}{\frac{2+2\sqrt{3}}{\sqrt{3}}}$

$= \dfrac{1}{\sqrt{2}} \times \dfrac{\sqrt{3}}{2+2\sqrt{3}} = \dfrac{\sqrt{3}}{\sqrt{2} \times 2\left(\sqrt{3}+1\right)}$

$$= \frac{\sqrt{3}}{\sqrt{2}\times 2\left(\sqrt{3}+1\right)} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$$

$$= \frac{\sqrt{3}\left(\sqrt{3}-1\right)}{\sqrt{2}\times 2(3-1)}$$

$$= \frac{\sqrt{3}\left(\sqrt{3}-1\right)}{4\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{3\sqrt{2}-\sqrt{6}}{8}$$

(iv) $$\frac{\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ}{\sin 30^\circ + \cos 45^\circ + \cot 45^\circ} = \frac{\frac{1}{2}+1-\frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}}+\frac{1}{2}+1}$$

$$= \frac{\frac{\sqrt{3}+2\sqrt{3}-4}{2\sqrt{3}}}{\frac{4+\sqrt{3}+2\sqrt{3}}{2\sqrt{3}}}$$

$$= \frac{3\sqrt{3}-4}{3\sqrt{3}+4}$$

$$= \frac{3\sqrt{3}-4}{3\sqrt{3}+4} \times \frac{3\sqrt{3}-4}{3\sqrt{3}-4}$$

$$= \frac{27+16-24\sqrt{3}}{27-16} = \frac{43-24\sqrt{3}}{11}$$

(v) $$\frac{5\cos^2 60^\circ + 4\sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ}$$

$$= \frac{5\left(\frac{1}{2}\right)^2 + 4\left(\frac{2}{\sqrt{3}}\right)^2 - (1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \frac{5\times\frac{1}{4} + 4\times\frac{4}{3} - 1}{\frac{1}{4} + \frac{3}{4}}$$

$$= \frac{\frac{1}{12}\times 67}{\frac{4}{4}}$$

$$= \frac{67}{12}$$

8. Prove the following identities, where the angles involved are acute angles for which the expressions are defined:

(i) $(\operatorname{cosec}\theta - \cot\theta)^2 = \frac{1-\cos\theta}{1+\cos\theta}$

(ii) $\frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A} = 2\sec A$

(iii) $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1+\sec\theta\operatorname{cosec}\theta$

(iv) $\frac{1+\sec A}{\sec A} = \frac{\sin^2 A}{1-\cos A}$

(v) $\dfrac{\cos A-\sin A+1}{\cos A+\sin A-1}=\operatorname{cosec} A+\cot A$, **using the identity** $\operatorname{cosec}^2 A=1+\cot^2 A$

(vi) $\sqrt{\dfrac{1+\sin A}{1-\sin A}}=\sec A+\tan A$

(vii) $\dfrac{\sin\theta-2\sin^3\theta}{2\cos^3\theta-\cos\theta}=\tan\theta$

(viii) $(\sin A+\operatorname{cosec} A)^2+(\cos A+\sec A)^2=7+\tan^2 A+\cot^2 A$

(ix) $(\operatorname{cosec} A-\sin A)(\sec A-\cos A)=\dfrac{1}{\tan A+\cot A}$

(x) $\left(\dfrac{1+\tan^2 A}{1+\cot^2 A}\right)=\left(\dfrac{1-\tan A}{1-\cot A}\right)^2=\tan^2 A$

Ans. (i) L.H.S. $(\operatorname{cosec}\theta-\cot\theta)^2$

$=\operatorname{cosec}^2\theta+\cot^2\theta-2\operatorname{cosec}\theta\cot\theta$

$=\dfrac{1}{\sin^2\theta}+\dfrac{\cos^2\theta}{\sin^2\theta}-2\times\dfrac{1}{\sin\theta}\cdot\dfrac{\cos\theta}{\sin\theta}$

$=\dfrac{1+\cos^2\theta}{\sin^2\theta}-\dfrac{2\cos\theta}{\sin^2\theta}$

$=\dfrac{1+\cos^2\theta-2\cos\theta}{\sin^2\theta}$

$=\dfrac{(1-\cos\theta)^2}{\sin^2\theta}$ $\left[\because a^2+b^2-2ab=(a-b)^2\right]$

$=\dfrac{(1-\cos\theta)(1-\cos\theta)}{1-\cos^2\theta}$

$$= \frac{(1-\cos\theta)(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{1-\cos\theta}{1+\cos\theta} = \text{R.H.S.}$$

(ii) L.H.S. $\dfrac{\cos A}{1+\sin A} + \dfrac{1+\sin A}{\cos A}$

$$= \frac{\cos^2\theta + 1 + \sin^2\theta + 2\sin A}{(1+\sin A)\cos A}$$

$$= \frac{\cos^2\theta + \sin^2\theta + 1 + 2\sin A}{(1+\sin A)\cos A}$$

$$= \frac{1+1+2\sin A}{(1+\sin A)\cos A} \quad \left[\because \sin^2\theta + \cos^2\theta = 1\right]$$

$$= \frac{2+2\sin A}{(1+\sin A)\cos A} = \frac{2(1+\sin A)}{(1+\sin A)\cos A} = \frac{2}{\cos A}$$

$= 2\sec A$ = R.H.S

(iii) L.H.S. $\dfrac{\tan\theta}{1-\cot\theta} + \dfrac{\cot\theta}{1-\tan\theta}$

$$= \frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}}$$

$$= \frac{\sin\theta}{\cos\theta} \times \frac{\sin\theta}{\sin\theta - \cos\theta} + \frac{\cos\theta}{\sin\theta} \times \frac{\cos\theta}{\cos\theta - \sin\theta}$$

$$= \frac{\sin^2\theta}{\cos\theta(\sin\theta - \cos\theta)} + \frac{\cos^2\theta}{\sin\theta(\cos\theta - \sin\theta)}$$

$$= \frac{\sin^2\theta}{\cos\theta(\sin\theta - \cos\theta)} - \frac{\cos^2\theta}{\sin\theta(\sin\theta - \cos\theta)}$$

$$= \frac{\sin^3\theta - \cos^3\theta}{\sin\theta\cos\theta(\sin\theta - \cos\theta)}$$

$$= \frac{(\sin\theta - \cos\theta)(\sin^2\theta + \cos^2\theta + \sin\theta\cos\theta)}{\sin\theta\cos\theta(\sin\theta - \cos\theta)}$$

$$\left[\because a^3 - b^3 = (a-b)(a^2 + b^2 + ab)\right]$$

$$= \frac{1 + \sin\theta\cos\theta}{\sin\theta\cos\theta} \quad \left[\because \sin^2\theta + \cos^2\theta = 1\right]$$

$$= \frac{1}{\sin\theta\cos\theta} + 1$$

$$= 1 + \frac{1}{\sin\theta\cos\theta}$$

$$= 1 + \sec\theta\operatorname{cosec}\theta$$

(iv) L.H.S. $\frac{1+\sec A}{\sec A} = \frac{1 + \frac{1}{\cos A}}{\frac{1}{\cos A}}$

$$= \frac{\cos A + 1}{\cos A} \times \frac{\cos A}{1} = 1 + \cos A$$

$$= 1 + \cos A \times \frac{1 - \cos A}{1 - \cos A}$$

$$= \frac{1 - \cos^2 A}{1 - \cos A}$$

$$= \frac{\sin^2 A}{1 - \cos A} = \text{R.H.S.}$$

(v) L.H.S. $\dfrac{\cos A-\sin A+1}{\cos A+\sin A-1}$

Dividing all terms by $\sin A$,

$= \dfrac{\cot A-1+\operatorname{cosec}A}{\cot A+1-\operatorname{cosec}A} = \dfrac{\cot A+\operatorname{cosec}A-1}{\cot A-\operatorname{cosec}A+1}$

$= \dfrac{(\cot A+\operatorname{cosec}A)-(\operatorname{cosec}^2A-\cot^2 A)}{(1+\cot A-\operatorname{cosec}A)}$

$= \dfrac{(\cot A+\operatorname{cosec}A)+(\cot^2 A-\operatorname{cosec}^2A)}{(1+\cot A-\operatorname{cosec}A)}$

$= \dfrac{(\cot A+\operatorname{cosec}A)(1+\cot A-\operatorname{cosec}A)}{(1+\cot A-\operatorname{cosec}A)}$

$= \cot A+\operatorname{cosec}A$ = R.H.S.

(vi) L.H.S. $\sqrt{\dfrac{1+\sin A}{1-\sin A}}$

$= \sqrt{\dfrac{1+\sin A}{1-\sin A}} \times \sqrt{\dfrac{1+\sin A}{1+\sin A}}$

$= \sqrt{\dfrac{(1+\sin A)^2}{1-\sin^2 A}}$ $[\because (a+b)(a-b)=a^2-b^2]$

$= \sqrt{\dfrac{(1+\sin A)^2}{\cos^2 A}}$ $[\because 1-\sin^2\theta=\cos^2\theta]$

$= \dfrac{1+\sin A}{\cos A} = \dfrac{1}{\cos A}+\dfrac{\sin A}{\cos A} = \sec A+\tan A$ = R.H.S.

(vii) L.H.S. $\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \frac{\sin\theta(1-2\sin^2\theta)}{\cos\theta(2\cos^2\theta - 1)}$

$= \frac{\sin\theta(1-2\sin^2\theta)}{\cos\theta\left[2(1-\sin^2\theta)-1\right]}$ $[\because 1-\sin^2\theta = \cos^2\theta]$

$= \frac{\sin\theta(1-2\sin^2\theta)}{\cos\theta(2-2\sin^2\theta-1)}$

$= \frac{\sin\theta(1-2\sin^2\theta)}{\cos\theta(1-2\sin^2\theta)} = \frac{\sin\theta}{\cos\theta} = \tan\theta$ = R.H.S

(viii) L.H.S. $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2$

$= \left(\sin A + \frac{1}{\sin A}\right)^2 + \left(\cos A + \frac{1}{\cos A}\right)^2$

$= \sin^2 A + \frac{1}{\sin^2 A} + 2.\sin A.\frac{1}{\sin A} + \cos^2 A + \frac{1}{\cos^2 A} + 2.\cos A.\frac{1}{\cos A}$

$= 2 + 2 + \sin^2 A + \cos^2 A + \frac{1}{\sin^2 A} + \frac{1}{\cos^2 A}$

$= 4 + 1 + \frac{1}{\sin^2 A} + \frac{1}{\cos^2 A}$

$= 5 + \operatorname{cosec}^2 A + \sec^2 A$

$= 5 + 1 + \cot^2 A + 1 + \tan^2 A$

$[\because \operatorname{cosec}^2\theta = 1 + \cot^2\theta, \sec^2\theta = 1 + \tan^2\theta]$

$= 7 + \tan^2 A + \cot^2 A$

= R.H.S.

(ix) L.H.S. $\left(\cos ecA - \sin A\right)\left(\sec A - \cos A\right)$

$= \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)$

$= \left(\frac{1-\sin^2 A}{\sin A}\right)\left(\frac{1-\cos^2 A}{\cos A}\right)$

$= \frac{\cos^2 A}{\sin A} \times \frac{\sin^2 A}{\cos A}$ $= \sin A . \cos A$

$= \frac{\sin A . \cos A}{\sin^2 A + \cos^2 A}$

$\left[\because \sin^2\theta + \cos^2\theta = 1\right]$

Dividing all the terms by $\sin A . \cos A$,

$= \frac{\frac{\sin A . \cos A}{\sin A . \cos A}}{\frac{\sin^2 A}{\sin A . \cos A} + \frac{\cos^2 A}{\sin A . \cos A}}$ $= \frac{1}{\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}}$

$= \frac{1}{\tan A + \cot A}$ = R.H.S.

(x) L.H.S. $\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right)$ $= \frac{\sec^2 A}{\cos ec^2 A}$

$\left[\because 1+\tan^2\theta = \sec^2\theta, 1+\cot^2\theta = \cos ec^2\theta\right]$

$= \frac{1}{\cos^2 A} \times \frac{\sin^2 A}{1}$ $= \tan^2 A$ = R.H.S.

Now, Middle side = $\left(\frac{1-\tan A}{1-\cot A}\right)^2$

$$= \left(\frac{1 - \tan A}{1 - \frac{1}{\tan A}} \right)^2$$

$$= \left(\frac{1 - \tan A}{\frac{\tan A - 1}{\tan A}} \right)^2$$

$$= \left(\frac{1 - \tan A}{\frac{-(1 - \tan A)}{\tan A}} \right) = (-\tan A)^2$$

$= \tan^2 A$ = R.H.S.